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Robust Portfolio Optimization Strategies in The Serbian Stock Market

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Abstract:

Research Question: This paper investigates the performances of six portfolios constructed using robust optimization methods in the Serbian stock market. **Motivation:** Motivated by the lack of research that analyses the allocation strategies based on robust optimization in the other non-US markets, this paper analyses the ability of these strategies to produce positive performance in the Serbian financial market. **Idea:** This paper aims to check whether robust strategies can provide positive risk-adjusted performance compared to simple strategies. **Data:** The analysis was performed on daily data from 2017 to 2020. **Tools:** We used monthly portfolio rebalancing with an estimation period of 24 months, applying budget and no-short selling constraints in portfolio construction. As a benchmark, we used two simple strategies, the strategy of market index replication and the equal weighting strategy (1/N). Consequently, the performance of the portfolios is evaluated once a month and calculated for the entire investment period. **Findings:** Empirical results suggest that robust optimization methods improve portfolio performance on a risk-adjusted basis. The increase in performance is affected by an increase in turnover, so the stability of weights in the portfolio depends on the compliance of the model characteristics with the conditions prevailing in the market. **Contribution:** To the best of our knowledge, this is the first article that analyses the performance of robust optimization portfolios for the Serbian stock markets. Analysing the performance of robust optimization strategies and comparing them to two simple strategies, this paper contributes to the existing literature by checking their possibility of obtaining a positive performance in less developed markets. Additionally, all information presented in this paper could help investors optimize their risk allocation and profitability.

Keywords: portfolio allocation, mean-variance optimization, robust estimation, estimation error, shrinkage estimators

JEL Classification: G11, G15

1. Introduction

The dominant influence in the theory of portfolio choice and investment decisions in recent decades had Markowitz's (Markowitz, 1952) theory of portfolio choice. This theory defined the process of making investment decisions in a portfolio context as a quantitative process that requires that the expected return and risk of security should be specified and considered together. Specification of expected values of parameters for securities means that these parameters have to be estimated, and the consideration of alternative investment choices should be based on risk-return trade-off.

The portfolio weights are highly sensitive to estimation errors, so an accurate forecast of inputs is of great importance in the forecasting process. Inaccurate inputs affect the optimization process in a way that solutions result in unintuitive and unstable portfolio weights with a small number of instruments in the portfolio. Consequently, the optimal portfolio's constitutive elements are susceptible to small changes in the input parameters. For example, these issues are documented by Chopra & Ziemba (1993) and Michaud (1989). The absence of a formal affirmation of Markowitz's concept in practical application is primarily the result of these

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weaknesses. Thus, the concept's practical implementation requires a reliable and robust model with a higher degree of resistance to estimation errors (Kolm et al., 2014). Hence, it is evident that the critical step in portfolio construction defined in this way is the accurate estimation of expected inputs. One way to solve the issue regarding sensitivity is to use estimators who are less sensitive to noise in the data. The second way is to determine the set as a confidence interval for the input parameters that contain all or most of the possible values of inputs. Regardless of how the sensitivity issue is solved in the optimization, the goal is to generate a robust solution that provides positive performance. Hence the common name for these methods is the robust optimization methods.

As documented in Kim et al. (2013b), literature concerning robust optimization mainly concentrates on formulating robust problems or the properties of robust portfolios, primarily using simulated or US stock market data. Motivated by the lack of research that analyses the allocation strategies based on robust optimization in the other non-US markets, this paper analyses the ability of these strategies to produce positive performance in the Serbian financial market. The investment environment Serbian market differs significantly from the US financial market. However, we intend to validate its advancement in reducing the sensitivity of portfolio weights and validate its ability to provide positive risk-adjusted performance.

We test the hypothesis that at least one robust strategy from the set of considered robust methods achieves better performance than two simple investment strategies, either on a Sharpe ratio basis or based on the Modigliani-Modigliani (M2) measure. We focus on the Serbian financial market in the 2017-2020 period.
Although the set of available robust optimisation models for portfolio allocation is wide, our research focuses on six robust allocation strategies based on the classical mean-variance model by analysing the daily data between 2017 and 2020. We show the properties of robust portfolios in this market and report their ability to solve the problem of weight sensitivity in this environment. Further, we compare the performance of robust portfolios against two simple strategies, replicated market index portfolio and a portfolio with equal weights (1/N). We use these strategies as a benchmark because of their ability to provide diversification without optimisation. We analyse the level of portfolio concentration and turnover and compare them based on a risk-adjusted basis using the mean-variance framework for investors with a long-only position and usual non-negativity and budget constraints.

DeMiguel et al. (2009) find that the performances of fourteen different models of optimal portfolio allocation strategies relative to the strategy which assumes equal weighting (1/N) are significantly lower. Opposite to these results, and more recently, Kim et al. (2018) found by analysing the US stock market from 1980 to 2014 that portfolios formed by robust portfolio optimisation produce solutions that reduce a worst-case loss. These abilities make them efficient relative to the market index portfolio, the equally-weighted portfolio, and the global minimum variance portfolio. However, there is a significant difference in the development level and the available number of securities between the analysed market and the US stock market, and this paper tests these results to see if this is the case with financial markets with lower development level and a low number of securities for trading. Hence, the paper's main contribution is to examine the performance of robust optimisation strategies and compare them to two simple strategies, such as an equal-weighted portfolio and replicated index portfolio, in the frontier financial market, such as the Serbian financial market. To the best of our knowledge, this is the first article that analyses the characteristics of robust optimisation portfolios on data for non-US countries to check whether it is possible to obtain positive performance in markets with a low number of securities. Although the performance may be affected to some degree by the selected data set and period, overall empirical results suggest that robust optimisation methods improve the portfolio performance on a risk-adjusted basis. The increase in performance is affected by an increase in turnover. Consequently, the portfolio's robustness depends on the model characteristics' compliance with the market's prevailing conditions.

The rest of the paper is structured as follows. Section two shows the review of literature related to robust portfolio optimization. Section three shows the information about the dataset and applied methodology. Section four shows the performance measures used in the analysis. The results and the interpretations of our findings are presented in Section 5. The final section presents the conclusion.

2. Literature Review

Harry Markowitz was the first author to introduce a systematic approach to the asset allocation problem under uncertainty (Markowitz, 1959). Ideas presented in this article have highly impacted other research in financial economics related to portfolio choice and investment decisions since its publication. At its core, the investor decision-making process requires quantifying the expected return and risk of assets. Also, considering the interplay between risk and returns is required to get a simple analytical solution. Because the return and the risk are two opposite variables in the optimization process, the problem is formulated so that the expected return is maximized for any given level of portfolio risk. Alternatively, the portfolio variance is minimized for any given level of expected return. Despite the concept simplicity, the main problem arises from actual values of the expected returns; risks and covariances still need to be discovered. Thus, the practical implementation of this concept implies estimation or forecasting, which means that the whole process is exposed to estimation errors. These estimation errors significantly impact the portfolio weights, resulting in portfolios with high concentration or not well diversified. DeMiguel et al. (2009) testify that 1/N portfolios often perform better than mean-variance portfolios. Motivated by the limitation of the standard approach to MVO, other researchers try to modify the classical framework to achieve a stable and robust model concerning estimation errors. In general, there are two directions of action. One of them is aimed at finding more appropriate estimates of the parameters themselves, and the other way is to introduce improvements in the optimization process itself. The first direction implies the improvements in estimating expected returns and covariances by averaging different estimators, called shrinkage.

Shrinkage estimators are robust because they have low sensitivity to outliers and sampling errors. Jorion (1986) studied the effect of error estimation on portfolio choice and presented the type of James-Stein estimator for returns. The shrinkage target of this estimator is the return of the global minimum portfolio. He conducted a simulation analysis and showed that this type of shrinkage estimator significantly outperforms the classical sample mean in portfolio selection problems. Ledoit and Wolf (2003) multiply the sample covariance matrix by the single-index covariance matrix, thus making the covariance matrix of stock returns as a weighted average of two estimators. These authors found that the estimator formed by the average covariance matrix generated by Sharpe's (Sharpe W.F., 1963) single index model and historical covariance matrix can produce portfolios with a low level of risk even in comparison with other similar estimators in the relevant literature. In their later article, Ledoit and Wolf (2004) conclude that even better results are obtained by using the constant correlation model. This model assumes that all correlations' coefficient is the same for all portfolio assets. The idea is that instead of averaging the historical pair-wise correlation with the Sharpe single index model historical covariance matrix is combined with the constant correlation matrix or even by single usage of the matrix (Elton et al., 2006).

The Second direction refers to the improvements on the modelling side. That implies using constraints on portfolio weights, portfolio resampling, or implementing robust optimisation techniques. In the following, we will list only the articles related to the research methodology applied in this article. Jagannathan and Ma (2003) suggest the usage of global minimum variance portfolios (GMV) because the error in the estimation of the mean is so significant that they conclude that the loss incurred by ignoring the mean is minimum.

Clarke et al. (2011) also found that the GMV portfolio with long-only constraints in the U.S. market over the 1968–2009 period achieved a higher Sharpe ratio (0.45) compared to the market portfolio (0.35) and two other GMV portfolios, one without long-only constraints (0.42), and second based on single index model (0.43). Bastin (2015) also found that the portfolio with minimum variance outperformed the market index from 2006 to 2013 in the Czech stock market. Later, Bastin (2017) compared the performances of minimum variance portfolios to the performances of three types of equally-weighted portfolios and CDAX market index portfolios in the German stock market from 2002–2015. He confirms that the risk minimisation strategy is superior to other strategies. Besides placing constraints on portfolio weights, investors can use robust optimisation techniques to incorporate uncertainty into the optimisation process. That implies forming a set of values for parameters or generating scenarios. Robust portfolio optimisation differs from the traditional approach because it does not treat the inputs as deterministic. The robust approach assumes that inputs have been estimated with errors, so inputs are formulated as uncertainty sets that are made of possible forecasted point estimates (Xidonas et al., 2020). Formulating an uncertainty set assumes that the investor will maximise its utility, assuming that the worst-case values of the input parameters are realised from the uncertainty set. The investor's objective in this environment is to select a portfolio with expected returns and covariances that have maximum utility function, assuming the realisation of the most undesirable case of parameter values (Goldfarb & Iyengar, 2003). A comprehensive overview of the implementation of robust optimisation in portfolio selection is provided by Fabozzi et al. (2007) and Pachamanova et al. (2016). Xidonas et al. (2020) referenced 148 research works in recent 25 years and thus showed that robust optimisation has been very popular. However, Kim et al. (2014) documented that robust optimisation literature mainly concentrates on formulating robust problems or on the properties of robust portfolios, primarily using simulated or U.S. stock market data. Thus, to our knowledge, this is the first article that analyses the characteristics of robust optimisation portfolios on a small dataset for non-US countries.

Kim, J.H. et al. (2013a) confirm that solutions of the robust portfolio optimisation result in a lower number of asset classes than other portfolios constructed in the classical optimisation framework. More recently, Kim J.H. et al. (2018) documented the benefits of robust portfolio optimisation by analysing the performance of robust portfolios from 1980 to 2014 in the U.S. market. The authors documented that the performance of these portfolios relative to the global minimum variance portfolio, replicated market index portfolio, and equally weighted portfolio are superior on a risk-adjusted basis. Motivated by these results, we applied the same methodology concerning the uncertainty sets for robust strategies to see whether these results hold for the Serbian market considered in our analysis.

3. Data and Methodology

In order to test the performances of different portfolio allocation strategies, we analyse the financial market of Serbia using daily data on closing prices and stock returns. The small number of financial instruments and the low liquidity of the traded securities is the main feature of the Serbian financial market. In addition to the above, it is essential to note that a low level of trading is often accompanied by an irregular frequency of trading periods. Minovic and Zivkovic (2010) and Minovic (2017) documented these features.

As a potential investment universe, we analyse the most liquid stocks that are constituents of market indices, the BELEX15. The BELEX15 index measures the performance of the most liquid shares in the Serbian stock market. It is weighted only by free-float market capitalisation and serves as a benchmark to compare potential investment strategies according to its methodology. However, because the index measures the performances of the most liquid segment of the Serbian capital market and a significant portion of the whole market is persistently illiquid, it is not a perfect proxy for the entire market. Also, the number of elements in the composition of the BELEX15 index has decreased from its initial formation until today. Starting from the initial 15 securities in the basket composition, its basket today consists of 10 securities. Detailed information about the datasets used in this paper is in Table 1.

Table 1: Dataset information

Description:	Serbia
Investment universe	10
Market represented:	BELEX15
Maximum weighting cap %	20%
Index components	10
Risk free rate	T-bill:1 year
Number of observations in the test samples (OS)	1006
Sample structure:	2 years-Estimation period +1 month-Out-of-the-sample period
Total number of samples	48
Sample rotation:	1 month
Out-of-the-sample period:	1/Jan/2017 --- 31/Dec/2020 - 4 years

Data sets are obtained from the Thomson Reuters database and contain data from January 1, 2015, to December 31, 2020. We performed the analysis in the paper using monthly portfolio rebalancing with 24 months for estimation. Hence, the out-of-sample period ranges from January 1, 2017, to December 31, 2020, with a total of 48 months. We form optimal portfolios at each rebalancing period using the daily returns in the estimation period. The portfolios' performances are evaluated monthly and analysed during the entire investment period. Our methodological approach to sample rotation is identical to that of Kim et al. (2018) and Ledoit and Wolf (2017). According to these authors, monthly updating is an adequate frequency because it reduces the amount of excessive turnover and consequently reduces transaction costs. Therefore, we assume this simulates an actual investment situation appropriate for testing robust portfolios' sensitivity to market changes.

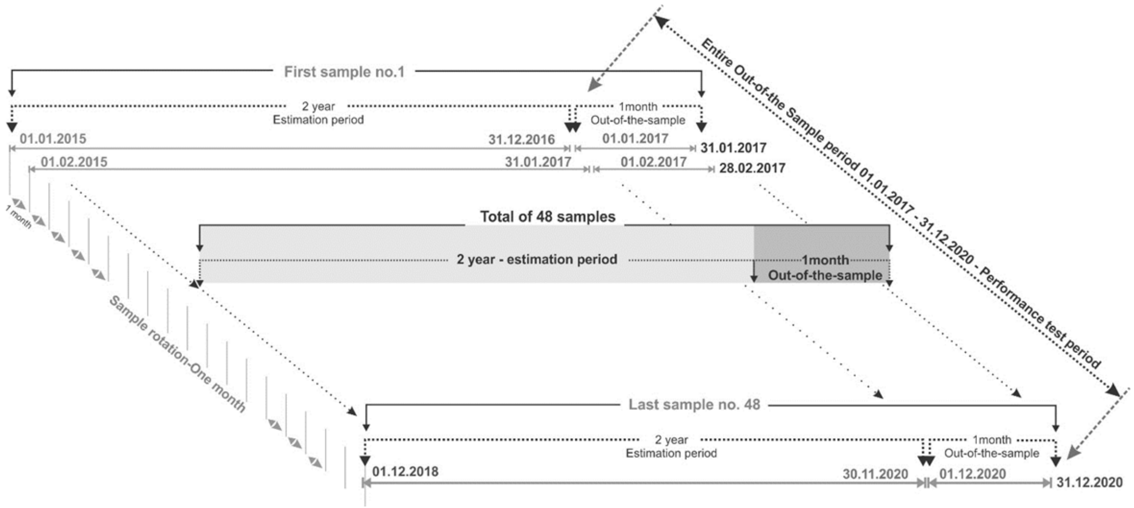

Figure 1: Sample rotation methodology

The basic framework for all strategies assumes Markowitz's (1952) approach, defined as a maximum expected return for a targeted expected risk level:

$$\max_w \ \mu' w - \delta w' \sum w \qquad (1)$$
$$s.t. \quad w' I = 1; \ 0 \leq \ w \leq 1, \qquad I' = [1, \ldots, 1].$$

Parameter $\mu = (\mu_1, \ldots, \mu_N)$ denotes expected returns while $\boldsymbol{w}$ is the vector of portfolio weights w= $(w_1, w_2, \ldots. w_N)$. The weight $\boldsymbol{w_i}$ is the fraction of the portfolio invested in stock *i*. The return covariance matrix with dimensions N×N is denoted as **Σ**, and parameter δ represents the coefficient of investor risk aversion. The covariance matrix Σ constitutes covariances between asset *i* and asset *j* marked as σ_{ij} where $\sigma_{ii} = \sigma_i^2$:

$$\sum = \begin{bmatrix} \sigma_{11} & \cdots & \sigma_{1N} \\ \vdots & \ddots & \vdots \\ \sigma_{N1} & \cdots & \sigma_{NN} \end{bmatrix} \qquad (2)$$

Return of a portfolio has been defined as a random variable with expected return μ_p and a variance σ_p^2:

$$\mu_p = w' \mu, \ \sigma_p^2 = w' \Sigma w \qquad (3)$$

In the optimization process for all strategies in the paper, we use two commonly used constraints: no negativity constraint ($0 \leq$ w≤ 1) and budget constraint $w' I = 1$. The allocation strategy, which follows the solutions of the mean-variance optimization defined by equation 1 in the rest of the paper, is marked with the abbreviation HIST. Standard mean-variance optimization defined by equation 1 could also be presented in the form of a risk minimization problem. In this formulation, the portfolio variance is minimized subject to the expected portfolio return: μ_t as a target:

$$\min_w \ \frac{1}{2} \sum_{i,j=1}^{N} w_i w_j \sigma_{ij} \qquad (4)$$
$$s.t. \ \sum_{i=1}^{N} w_i \mu_i = \mu_t \quad i \quad \sum_{i=1}^{N} w_i = 1$$

Solutions to all optimization problems are generated using the method of Lagrange multipliers:

$$L = \frac{1}{2} \sum_{i,j=1}^{N} w_i w_j \sigma_{ij} - \lambda_1 \left(\sum_{i=1}^{N} w_i \mu_i - \mu_t \right) - \lambda_2 \left(\sum_{i=1}^{N} w_i - 1 \right) \qquad (5)$$

Solving the problem defined in such a way results in a portfolio with a minimum variance on an efficient frontier. Assuming that covariance matrix Σ of assets returns is given, the portfolio generated by optimization with the lowest return variance is called the global minimum variance portfolio (GMV). The weights of this portfolio do not depend on the expected returns but only on the return variances and covariances (Memmel & Kempf, 2006). Jagannathan and Ma (2003) reported that estimation errors significantly influence the sample's mean. Hence the minimum variance portfolios generated by the sample covariance matrix with a forbidden short-sale as input perform almost similarly to ones caused by factor models or shrinkage estimation for risk assessment. Since we use shrinkage as a method to deal with parameter uncertainty and that placing the constraints of short selling or setting the maximum value for portfolio weights in the optimisation problem can be viewed as a form of shrinkage, we adopt the same methodology as Jagannathan and Ma (2003) in formulating a strategy of a global minimum variance portfolio. Solutions to the problem:

$$\min_w \frac{1}{2} w' \sum w \tag{6}$$
$$s.t. \quad w'I = 1;\ 0 \leq \ w \leq 1, \qquad I' = [1, \dots, 1].$$

represent the allocation strategy of the global minimum variance portfolio with forbidden short sales and the budget constraint. We will call it GMV throughout the rest of the paper.

In addition to this strategy, we analysed three more strategies based on shrinkage methods which stress the importance of better estimating the risk and the returns. The first strategy uses Jorion's estimator as representative of the expected return, and the second strategy uses the method with the constant correlation covariance matrix as the approximation for the covariance matrix instead of a historical covariance matrix. The third strategy uses the combination of the Jorion James Stein estimator and constant correlation covariance matrix methods to predict expected values of returns and risk, respectively.

Table 2: List of portfolio strategies in research

No.	Method for portfolio construction:	Category	Reference:	Strategy Abbreviation
1	Markowitz - The sample mean and covariance matrix.	Standard approach	Markowitz, (1952)	HIST
2	Global minimum variance portfolio (GMV) with long only constraints.	Shrinkage by constraints	Jagannathan and Ma (2003), Clarke et al. (2011), Bastin (2015), Bastin (2017)	GMV
3	The James Stein estimator with shrinkage target for returns: The mean of GMV – Jorion's Estimator.	Shrinkage estimation	Jorion (1986)	JOR
4	The sample mean and constant correlation covariance matrix for risks approximation.	Shrinkage estimation	Ledoit and Wolf (2003)	CC
5	The James Stein Estimator for returns and constant correlation covariance matrix for risks approximation	Shrinkage estimation	Jorion (1986), Ledoit and Wolf (2004)	CCJS
6	Robust optimization with the "Box" uncertainty set.	Uncertainty included in optimization problem	Kim et al. (2018), Fabozzi et al. (2007), Tutuncu and Koenig (2004)	RBOX
7	Robust optimization with the "Ellipsoid" uncertainty set.	Uncertainty included in optimization problem	Kim et al. (2018), Fabozzi et al. (2007), Kim et al. (2014)	RELPS
8	Equal weight for all stocks in portfolio.	Simple allocation	DeMiguel (2009)	1/N
9	Market portfolio replicated by index.	Simple allocation		M

Shrinkage is the method based on averaging different estimators into one (Fabozzi et al. 2007). In this paper we use the Jorion Estimator (Jorion, 1986), a type of estimator which takes the return of the global minimum portfolio μ_{GMV} as a shrinking target and I = [1,1, ..., 1]'.

$$\widehat{\mu_{JS}} = (1 - w)\hat{\mu} + w\mu_{GMV} I \tag{7}$$

The return of this portfolio μ_{GMV} without the restriction on short-sales is defined as:

$$\mu_{GMV} = \frac{I'\Sigma^{-1}\hat{\mu}}{I'\Sigma^{-1}I} \tag{8}$$

where $\hat{\mu}$ represents the sample mean and Σ is covariance matrix generated from the sample. Shrinkage coefficient *w* is defined as:

$$w = \frac{N+2}{N+2+T(\hat{\mu}-\mu_{GMV}I)'\Sigma^{-1}(\hat{\mu}-\mu_{GMV}I)} \tag{9}$$

Parameter *N* represents the number of securities in portfolio and the parameter *T* is the total number of observations in the estimation period. The third optimization problem with the Jorion estimator from Table 2 is formulated as:

$$\max_w \ \widehat{\mu_{JS}}' w - \delta w' \sum w \tag{10}$$
$$s.t. \quad w'I = 1; \ 0 \leq \ w \leq 1, \qquad I' = [1, \dots, 1].$$

In the rest of the paper, the solutions (*w*) to equation 10 will present the allocation strategy that uses the Jorion estimator μ_{JS} for expected returns and the sample covariance matrix $\boldsymbol{\Sigma}$ for expected risks approximation. We will use the abbreviation JOR when we present the performance of this strategy. Also, in the group of methods that belong to shrinkage estimation, we used covariance matrix estimation based on the constant correlation model suggested by Ledoit and Wolf (2004), which assumes the same pairwise correlations for all stocks. The estimator of the common constant correlation coefficient is the arithmetic mean of all the sample correlation coefficients. This number forms a shrinkage target matrix with the vector of sample variances. The estimator for the covariance matrix that is calculated as a shrinkage is:

$$\widehat{\Sigma_{LW}} = \delta \widehat{\Sigma_{CC}} + (1 - \delta)\hat{\Sigma} \tag{11}$$

where δ represents a shrinkage constant, $\widehat{\Sigma_{CC}}$ is a sample covariance matrix with one constant correlation coefficient, and $\hat{\Sigma}$ is a sample covariance matrix. Since authors confirm that similar results are achieved if the historical covariance matrix is shrunk toward the constant correlation matrix or by single usage of the matrix, which assumes that all correlations are the same (Elton et al. (2006)), we use δ=1, so the shrinkage estimator for covariance matrix in our case equals to the constant correlation matrix $\widehat{\Sigma_{CC}}$. In order to calculate the constant correlation matrix for *N* securities in the portfolio, the sample covariance matrix must be decomposed as a product of the diagonal matrix of returns volatilities $\boldsymbol{R}$ and sample correlation matrix $\boldsymbol{C}$:

$$\hat{\boldsymbol{\Sigma}} = \boldsymbol{RCR'}$$

Then we replace all individual sample correlation coefficients $\hat{\rho}_{ij}$ in matrix **C** by an average constant correlation coefficient $\hat{\rho}$:

$$\hat{\rho} = \frac{2}{(N-1)N} \sum_{i=1}^{N} \sum_{j=I+1}^{N} \hat{\rho}_{ij} \tag{12}$$

$$C = \begin{bmatrix} 1 & \hat{\rho}_{12} & \cdots & \hat{\rho}_{1N} \\ \hat{\rho}_{21} & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \hat{\rho}_{N-1N} \\ \hat{\rho}_{N1} & \cdots & \hat{\rho}_{NN-1} & 1 \end{bmatrix}, C_{CC} = \begin{bmatrix} 1 & \hat{\rho} & \cdots & \hat{\rho} \\ \hat{\rho} & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \hat{\rho} \\ \hat{\rho} & \cdots & \hat{\rho} & 1 \end{bmatrix}$$

$$\widehat{\boldsymbol{\Sigma_{CC}}} = \boldsymbol{R C_{CC} R'}.$$

The fourth method from Table 2 represents the problem with the sample mean $\hat{\mu}$ and constant correlation covariance matrix $\widehat{\Sigma_{CC}}$ for risks is formulated as:

$$\max_w \ \hat{\mu}' w - \frac{1}{2} w' \widehat{\Sigma_{CC}} w \tag{13}$$
$$s.t. \quad w'I = 1; \ 0 \leq \ w \leq 1, \qquad I' = [1, \dots, 1].$$

The solutions (*w*) to the equation 13 will present the weights for allocation strategy that uses the sample mean $\hat{\mu}$ as estimator for expected returns and the sample constant correlation covariance matrix $\widehat{\Sigma_{CC}}$ as estimator for expected risks will be abbreviated as CC in the rest of paper.

The fifth method combines shrinkage estimators that we used in the case of the Jorions estimator (JOR) and the estimator of constant correlation covariance matrix (CC), thus forming the model numbered as 5 in Table 2. Soloutions to this optimization problem:

$$\max_w \ \widehat{\mu_{JS}}'w - \frac{1}{2}w'\widehat{\mathbf{\Sigma}_{CC}}w \qquad (14)$$
$$s.t. \quad w'I = 1;\ 0 \leq \ w \leq 1, \qquad I' = [1, \ldots, 1].$$

that is based on two shrinkage estimators represents the allocation strategy abbreviated as CCJS in the rest of the paper.

In addition to shrinkage estimators, we use two robust methods that directly include parameter uncertainty in optimisation problems. Assuming the confidence region for returns is set in advance with other constraints, those methods try to find the maximum of a function by assuming the realisation of the lowest expected return. Starting from the fact that estimation error is more dominant in returns than in variances and covariances (Kan & Zhou, 2007), we use two robust methods that incorporate the uncertainty of returns in optimisation problems. The first is known as the box uncertainty set, and the second is the ellipsoidal uncertainty set. Fabozzi et al. (2007) present detailed explanations and formulations of these sets. The difference between these two sets is that in the case of box uncertainty, we define the expected return for each asset by the individual interval of possible values. In contrast, interval refers to the joint combined set for the expected return vector in the ellipsoidal set. However, we adopt the same methodology concerning these sets as in (Kim et al., 2018), so the box uncertainty in this paper assumes that a normal distribution characterises the stock returns distribution with a 95% confidence interval around the estimate of expected return.

$$Ubox_d(\widehat{\mu}) := \{\mu : |\mu_i - \hat{\mu}_i| \leq d_i\,, i = 1, \ldots, N\} \qquad (15)$$

where $d_i = 1.96\sigma_i/\sqrt{T}$, and T is the number of observations. In our case, the sample size is long enough (T>500) to use this assumption according to the Central Limit Theorem. Parameter $\hat{\mu}_i$ is an average estimate of individual returns μ_i, and $d_i$represent the potential deviation of return for the individual security from its estimated value. That means that estimation error from estimating expected return is not bigger than d_i; thus the main optimization problem is:

$$\max_w \left\{\min_{\mu \in Ubox_d(\widehat{\mu})} \mu'w - \frac{1}{2}w'\Sigma\mathrm{w}\right\} \ s.t.\ w'I = 1,\ I = \{1, \ldots, N\} \qquad (16)$$

and its robust formulation that we solve is:

$$\max_w \hat{\mu}'w - \frac{1}{2}w'\Sigma\mathrm{w} - d'|w| \qquad s.t.\ w'I = 1,\ ;\ 0 \leq \ w \leq 1, I = \{1, \ldots, N\} \qquad (17)$$

The second uncertainty set is defined in such a way that the scaled sum of constitutive assets returns is no higher than h^2, thus forming a joint confidence region with an elliptical shape. We use historical data during the estimation period to construct the uncertainty set. We set the size of the confidence interval to 95% with the assumption of normality, so the probability of the true returns inside this set of ellipsoidal shapes depends on the χ^2 distribution (Goldfarb & Iyengar, 2003). The number of assets that constitute the investment universe for a particular market is used as the number of degrees of freedom to obtain critical values of the Chi-square distribution for the probability that the critical value of α=0.05 will be exceeded. The Ellipsoidal uncertainty set is defined as:

$$Uelps_\delta(\hat{\mu}) := \left\{\mu : (\mu - \hat{\mu})' \textstyle\sum_\mu^{-1} (\mu - \hat{\mu}) \leq h^2 \right\} \qquad (18)$$

and the main optimization problem takes the form:

$$\max_w \left\{\min_{\mu \in Uelps_\delta(\widehat{\mu})} \mu'w - \lambda w'\Sigma\mathrm{w}\right\} s.t.\ w'I = 1,\ I = \{1, \ldots, N\} \qquad (19)$$

whose robust formulations is defined as:

$$\max_w \ \hat{\mu}'w - \frac{1}{2}w'\Sigma\mathrm{w} - h\sqrt{w' \textstyle\sum_\mu w} \quad s.t.\ w'I = 1,\ ;\ 0 \leq \ w \leq 1, I = \{1, \ldots, N\} \qquad (20)$$

Robust portfolios constructed of solutions to equation 17 are abbreviated as RBOX strategy. The portfolios constructed of solutions to equation 20 are abbreviated as the RELPS strategy in the remaining of the paper.

4. Performance Measures

To analyse the performance of different robust portfolios and compare them to the performance of two portfolios based on simple strategies, we used the set of measures that refers to the return, risk, and structure of portfolios. Performances of portfolios are evaluated monthly and collected for the entire out-of-sample period ranging from January 1, 2017, to December 31, 2020, with a total of 48 months. Besides the average realised holding return (HRP) and risk (RR) of the portfolio, we use a set of seven measures which includes the Sharpe ratio (SR), Modigliani-Modigliani risk-adjusted measure (M2), the terminal wealth of portfolio (TW), Herfindahl-Hirschman index (HHI), the inverse of the Herfindahl-Hirschman index (1/HHI), portfolio turnover (TOR) and average absolute number of shares subject to change in portfolio structure due to rebalancing between two periods (AANS).

Since we test the hypothesis that at least one robust strategy achieves a better performance of two simple investment strategies on a single market either on a Sharpe ratio basis or on the basis of Modigliani-Modigliani measure, we show here how these measures are defined. The Sharpe ratio shows an average excess return of the portfolio over the risk-free rate, per unit of portfolio risk:

$$SR_{i,p} = \frac{\bar{r}_{i,p} - \bar{r}_{i,f}}{\sigma_{i,p}}. \tag{21}$$

Parameter $\bar{r}_{i,p}$ refers to the realized portfolio return in month *i* and $\bar{r}_{i,f}$ represents the average risk-free rate in month *i*, while parameter $\sigma_{i,p}$ represents the realized volatility of the portfolio *p* in month *i*. The Sharpe ratio is a risk-adjusted performance measure that measures the trade-off between risk and returns and shows the mean-variance efficiency of the portfolio under consideration. Because of the denominator of the Sharpe ratio, which takes individual risks, it is not desirable to compare the different strategies based solely on this indicator. Proper comparison requires considering that differences between the Sharpe ratios of different strategies could not be quantified because of the distinct denomination. Because we compare performances of robust portfolios relative to market performances, we use the Modigliani-Modigliani M2 performance measure that is adjusted for risk and presents a linear transformation of the Sharpe ratio (Modigliani L. & Modigliani F., 1997). The M^2 measure helps us identify the intensity of performance differences between alternative approaches by adjusting the risk of the portfolio to the market risk, which is a benchmark in our case. Adjusting of portfolio risk is achieved by combining Treasury bills with the portfolio holdings. To calculate the M^2 measure we use the following formula:

$$M^2 = \frac{\bar{r}_p - \bar{r}_f}{\sigma_p} \times \sigma_{market} + \bar{r}_f, \tag{22}$$

where σ_{market} represents the risk of market portfolio which we use as benchmark in our case. Since the M2 performance measure is a modified Sharpe ratio that makes the performance of different strategies comparable, because of the same denomination, we use it primarily as a criterion for qualifying the successfulness of a particular strategy in the analysed set of strategies for a particular market.

In the set of indicators that we used to check the performance of considered strategies, we use the indicator of terminal wealth. This indicator shows the cumulative portfolio value by particular strategy at the end of the out-of-sample period (31.12.2020) under the assumption of an initial investment of 1000 monetary units.

To compare applied methods based on transaction costs, we calculate a measure of portfolio turnover. This measure could be used as an approximation of transaction costs incurred by portfolio rebalancing because it measures the portion of the portfolio with *N* securities that is bought or sold over some period *T*. During the process of estimation, we use the definition of the indicator as DeMiguel et al. (2009):

$$Turnover(TOR) = \frac{1}{T}\sum_{t=1}^{T}\sum_{j=1}^{N}\left(\left|\widehat{w}_{i,t+1} - \widehat{w}_{i,t}\right|\right), \tag{23}$$

Asset *i* have a portfolio weight $\hat{w}_{i,t+1}$ at time *t+1*, after rebalancing the portfolio, and weight $\hat{w}_{i,t}$ before rebalancing in time *t*. A desirable property for the portfolio method would be the one that results in a lower value of this indicator, because this implies low transaction costs and a stable solution which requires a low level of rebalancing for the observed time frame.

5. Empirical Results

Following the presented methodology, the next section shows results for the Serbian financial market.

Table 3: Serbian market: Average monthly portfolio performance by allocation strategies in period from January 2017 to December 2020.

Strategy:	RBOX	RELPS	JOR	CC	CCJS	GMV	HIST	1/N	M
Method:	Robust optimization methods:						Standard	Simple allocation	
Improvement:	Optimization problem formulation		Parameter estimation-Shrinkage				-		
HRP:	-0.62	0.10	0.44	0.24	0.12	0.14	-0.07	0.11	-0.056
RR:	1.32	0.87	1.43	1.08	1.08	0.61	1.79	0.94	0.61
SR:	-0.47	0.11	0.30	0.22	0.10	0.21	-0.05	0.11	-0.10
M²:	-0.28	0.07	0.19	0.14	0.07	0.14	-0.02	0.08	-0.056
TW:	742	1048	1231	1123	1059	1068	965	1054	973
TOR:	27.6	14.8	18.6	7.43	0.95	4.3	19.1	-	-

The number of periods: 48 months; HRP (%)-Average holding return period for 1month, RR (%)-Average realized risk; SR-Sharpe ratio, the average risk-free rate in test period-T-bill: 0,01%; M²(%)-Modigliani-Modigliani risk-adjusted performance measure, TW-Terminal portfolio wealth-Assumed initial investment of 1000 monetary units; TOR (%)-Turnover Ratio-Average portion of the portfolio subject to change due to rebalancing;

Source: Author's calculation

Table 3 shows the average monthly portfolio performance for the applied allocation strategies in the financial market of Serbia. First, we report the results for the market index replication M strategy to see the general trend of market movement in the analysed period. Specifically, the index replication strategy achieved an average monthly decline in portfolio value, with a negative average value of the return of -0.056% per month, from the beginning of 2017 to the end of 2020. Assuming that the value of the replicated index portfolio is constantly declining at this rate, we see that the initial value of this portfolio lost 2.66% of its value in the four years. Translated into absolute terms, if we assume that the initial investment value at the beginning of 2017 was 1000 monetary units, the replicated portfolio will amount to 973.34 monetary units at the end of 2020. The average monthly realised return dispersion was 0.61%. Compared to other allocation strategies, the market portfolio and the GMV strategy that minimises the total portfolio risk achieved the lowest value of realised risk. The negative value of the realised return also caused a negative value of the Sharpe's ratio, so the average monthly value of this indicator was -0.10 in the observed period. Unlike the replicated index portfolio, the equal weighting strategy (1/N) achieved a positive return. The average realised value of the return of an equally weighted portfolio was 0.11%. The realised risk of this return was 0.94% on average, and the value of Sharpe's ratio was 0.11. Assuming the same risk for the equally weighted portfolio (1/N) and M portfolio, the average return for the 1/N strategy would be 0.08%, which is a significantly better result relative to the replicated market portfolio M.

The results are mixed regarding the overall performance of simple allocation strategies in the financial market of Serbia. For example, the portfolio strategy of following the structure of the market index achieved negative results, whereas the equal weighting strategy achieved overall positive results. Conversely, allocation strategies based on applying robust methods in total in terms of realised return achieved positive results since five out of six robust strategies achieved positive realised returns in the observed period.

The highest realised return in the analysed data set achieved the allocation strategy based on applying the Jorion estimator for estimating the expected return (JOR), whose return was 0.44% on average per month. The application of this strategy results in a cumulative return of 23.19% at the end of a four-year time interval. Assuming an initial investment of 1000 monetary units, the value of the portfolio constructed using optimisation based on the Jorion estimator for return approximation will amount to 1231.91 monetary units at the end of 2020. The variability of the realised return for this strategy was 1.43%, and compared to other strategies, we see that this is the strategy with the highest realised risk. Besides the significantly higher realised risk, the allocation of risk was relatively efficient since the average value of Sharpe's ratio for the JOR strategy is 0.30, which is the highest value in the considered allocation strategies. The superiority of the JOR strategy is also confirmed by a return of 0.19% on average per month if we assume that the average realised risk for the JOR strategy was equal to the risk of the replicated market portfolio M.

Compared to the adjusted return of the equally weighted portfolio (M2 = 0.08%), this value is more than twice as high. Based on Jorion's statistics, the portfolio structure consists of 8.25 securities on average in its composition, resulting in a small portfolio concentration measured by the HHI index (0.12). Since the level of concentration of the equally weighted portfolio (1/N) measured by the HHI index was 0.10, it is evident that the portfolio constructed using Jorion's estimator had almost the same level of diversification as the equally weighted portfolio. However, unlike the portfolio concentration indicator, which is favourable for the JOR strategy, the value of the indicator showing portfolio turnover indicates that it was necessary to change the portfolio structure by 18.60% each month to achieve the above-mentioned positive performance. Otherwise stated, the JOR strategy achieved positive performance by changing the portfolio's monthly composition by 1.53 securities on average.

Comparing the indicators of portfolio structure for the strategy with Jorion's estimator to the structure of the portfolio generated by other allocation strategies in the set, it is evident that positive performances were achieved with higher transaction costs. The higher level of transaction costs implicitly expressed through portfolio turnover indicators was achieved only by allocation strategies according to the standard approach HIST (19.16%) and robust allocation strategy with box uncertainty set RBOX (27.68%). In addition to the strategy with Jorion's statistics from a set of strategies based on robust methods concerning simple allocation approaches, more favourable results in terms of equivalent units of realized additional return measured by M2 measure were achieved by allocation strategies based on minimizing the total portfolio risk (GMV: 0.14%) and an allocation strategy that uses a matrix with an assumed constant correlation coefficient (CC: 0.14%) to assess expected risks. The terminal value of hypothetical portfolios according to these strategies (CC: 1123.42; GMV: 1068.04) is also higher than the equally weighted portfolio and the replicated index portfolio (1/N: 1054.65; M: 973.34). Although they have identical results in terms of performance measured by the M2 indicator, the level of diversification of these portfolios differed significantly in the observed period.

The GMV portfolio consisted on average of 1.45 securities, with a portfolio turnover ratio of 4.37%, which is the overall strategy with the lowest level of assumed transaction costs since its structure in terms of the number of securities remained unchanged, respectively robust. On the other hand, the strategy using a constant correlation matrix CC resulted in a portfolio with a high level of diversification (HHI: 0.10). Therefore, its structure changed more frequently (TOR: 7.43%) compared to GMV. Regardless, this strategy can be considered robust in terms of structure, as the portfolio structure of nine or ten securities (1/HHI: 9.94) changed by an average of 0.74 securities per month.

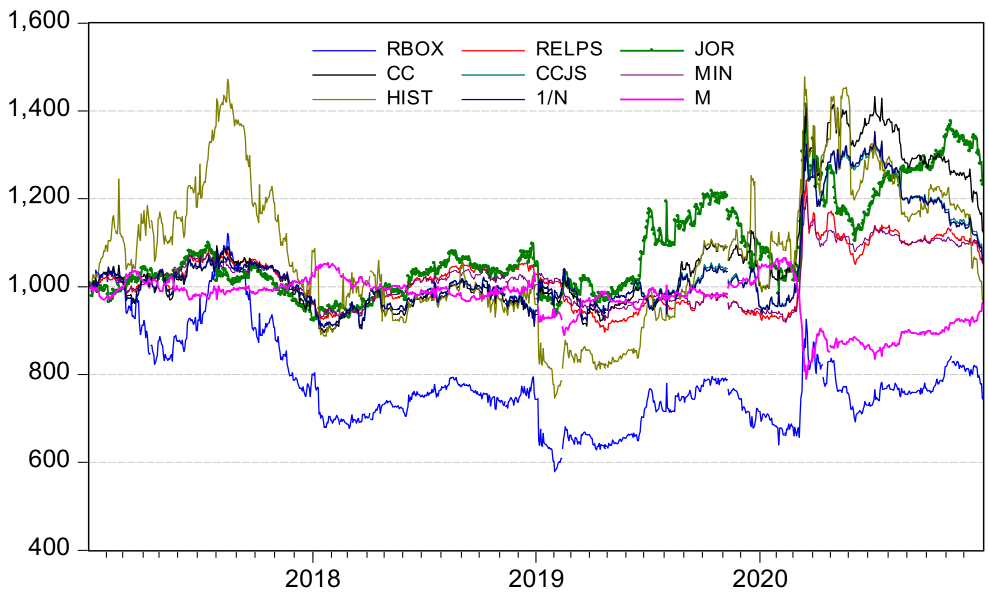

Figure 1: Serbian market - Cumulative portfolio value by different allocation strategy
Source: Author's estimation

Figure 2 shows the cumulative value of a hypothetical portfolio, assuming an adopted rebalancing methodology for the Serbian market. By visually observing Figure 2, we can divide the portfolio value fluctuation for all strategies into three segments. The first segment lasts from the beginning of 2017 until the first quarter of 2018. In this period, we can see the extreme movement of HIST and RBOX, but with diametrically different results. After this period starts the time segment in which the JOR approach made a significant step forward compared to other strategies. This period ranges from the first quarter of 2018 to the first quarter of 2020. Also, in this time interval, the portfolio's value, according to HIST, fluctuated significantly. The sharp decline in portfolio values characterises the end of 2020, caused by the COVID19 crisis and is a common feature of all approaches. Since it is evident that the intensity of oscillations in the movement of portfolio values is high during 2020, it follows that the crisis had a high impact on all strategies in the Serbian market. What is different is that the portfolio value according to the approach based on Jorion's estimator had the highest value growth and partial opposite reaction to adverse market movements in this time frame. Hence the portfolio value according to this approach had the highest terminal value at the end of 2020 in 1232 hypothetical monetary units.

Summarising the results for the financial market of Serbia, it is evident that robust strategies have achieved positive outcomes relative to the strategy of market replication. This result is opposite to the finding of DeMiguel et al. (2009) and in line with the findings of Kim et al. (2018) and Bastin (2017), with the caution that this result refers only to liquid shares included in the basket of the BELEX15 index. The best results in the financial market of Serbia, within the set of robust strategies, were achieved by strategies that account for the estimation risk by assessment of the expected values of return applying the shrinkage method. According to the results from Table 3, it is evident that the increase in return causes a significant increase in risk and the level of portfolio turnover. This pattern of movements is illustrated by jointly observing the performance of three robust approaches (JOR, CC, and GMV) with the best performance. Namely, the positive difference in the realised return for robust strategies JOR (0.44%) and the CC (0.24%) concerning the GMV return (0.14%) is significantly smaller than the difference in levels of realised risk for the strategies JOR (1.43%) and CC (1.08%) concerning the realised GMV risk (0.61%). In addition to the increase in investment risk in the observed strategies, the level of portfolio turnover increases as the values of this indicator for the strategies CC (7.43%) and JOR (18.60%) are significantly higher than one for the strategy of minimising the total risk of GMV (4.37%).

Analysing further the results of portfolio turnover indicators, it is noticeable that implementation of the strategy with frequent changes in the structure of the entire portfolio can result in significant losses for the investor. The above is supported by the results for the RBOX strategy with a turnover ratio of 27.68% on average per month, which, combined with the extremely high concentration (HHI: 0.90), implies that portfolio structure is changed completely approximately every three months. In this context, based on the analysed data set for the financial market of Serbia, it can be assumed that the relationship between portfolio turnover indicators and additional return per unit of risk for a particular strategy is of great importance for the generation of positive performance.

An empirically significant result of great importance for investors is that we identified the strategy with low turnover and a favourable holding return period. This optimisation strategy combines the Jorion estimator and the Matrix with a constant correlation coefficient for risk assessment. Regardless of the comparison in terms of adopted performance measures, CCJS strategy imply the lowest transaction cost with a robust structure, so its capabilities should be the subject of future research.

Further performance comparison of the GMV strategy and market portfolio shows that the risk minimisation strategy obtains better performance of the two strategies. Our result confirms the findings of Bastin (2015), who found that the minimum variance portfolio outperformed the PX index (market index) in the period from 2006 to 2013 in the Czech stock market. However, the author advises caution because the insufficient number of stocks in the Czech stock market constraints efficient diversification, suggesting that this stock market can serve only as a minor part of an investor's portfolio. This finding may also hold for the Serbian market. Alternatively, in line with the calculation of the M2 measure, which assumes the use of a risk-free rate to achieve equal risk levels, we can assume that a combination of risk-free securities and stocks can significantly improve the performance of the allocation strategy in the Serbian market.

Conclusion

Using the mean-variance framework, we examined the performance of six robust optimisation strategies in the Serbian financial market. We use the daily data from 2017 to 2020, using the rolling-sample approach with a 24 months estimation period. In all optimisation problems, we pose a short-selling constraint and the constraint that all weights must sum up to 1. Based on this methodology, we evaluate performances monthly and collect them for the entire four-year period. As a benchmark, we use the performances of two simple allocation strategies, the equal weighting strategy (1/N) and the strategy of market index replication.

Our empirical results suggest that robust optimisation methods improve portfolio performance on a risk-adjusted basis. Thus, we confirm our hypothesis because, from the set of considered robust methods in the observed period, there was at least one robust strategy with performances that are better than the performance of two simple investment strategies.

The results are confirmed either on a Sharpe ratio basis or the basis of the Modigliani-Modigliani M2 measure, which is more convenient for comparison. This result contrasts DeMiguel et al. (2009) and is generally in line with the findings of Kim et al. (2018), with the caution that these results refer to the US stock market. Since we analyse only the liquid segment of the financial market, we cannot generalise these results to the less liquid segment of the Serbian market. The increase in risk-adjusted performance caused a rise in turnover, so the stability of portfolio weights depends on the compliance of the model characteristics with the conditions prevailing in the market.

The presentation of the performance of a modern, robust approach to the portfolio selection problem, which can overcome the shortcomings of the standard portfolio selection approach, presents the main contribution of this paper. A robust approach makes it possible to make reliable assessments in portfolio selection by creating the basis for making optimal economic decisions, especially in a market with a small investment universe and low liquidity. Furthermore, we identified the strategy with low turnover and a favourable holding return period in the analysed period, which is an empirically significant result that could help all investors. This optimisation strategy that combines the Jorion estimator and the Covariance matrix with a constant correlation coefficient for risk assessment implies the lowest transaction cost with a robust structure, so its capabilities are interesting for future research.

Although the performance may be affected to some degree by the selected data set and period, it should be noted that the analysis in the paper refers to a single short period because of insufficient data. Further, the liquidity of underlying financial instruments is one of the factors that should have been taken into account in the construction of a portfolio because it could limit the execution of a particular strategy according to suggested rebalancing if the market is illiquid. Liquidity can be of great importance when rebalancing the portfolio with a significant concentration presupposes the inclusion or exclusion of security from a portfolio with insufficient volume. Additionally, in such cases, the effect of price impact that the realisation of a significant quantity of shares has on the price of a security was not considered, which can be especially important in markets where the number of trading instruments is small. Consequently, based on the available results, it is not desirable to make generalised conclusions about model properties. However, it is plausible to use them as a benchmark for a more extensive analysis in the future.

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